Unified, Unstructured Grids for Climate Modeling

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We have developed novel methods to generate unified, unstructured grids over the sphere with the goal of rendering the need to create separate grids for different applications obsolete and ushering in easier coupling of models while reducing coupling error. We create our spherical centroidal Voronoi tessellations and simultaneously their Delaunay duals with an iterative procedure. The most salient feature of our procedure is a complex density function that is able to take into account an arbitrary number of density proxies can represent physical measurements, model data, and/or error estimates. Thus, we can affect local refinement for different applications in different regions by way of multiple data sources. In addition, we take into account a representation of the shoreline in grid creation, allowing the triangulation to adapt itself to that representation to ensure that different applications will have appropriate interface regions.

The grids used to solve physical equations over the sphere, such as atmosphere and ocean models, have changed over the years and, moreover, the discretizations popular at a time inform the grid, and vice versa. A grid of latitude and longitude is the oldest and most widely used, marrying very naturally to both finite difference and finite volume methods. Over the last twenty years, in addition to seeing an increase in theory [1] and methodology to create a Voronoi diagram on the sphere, we have seen developments in grid generation and analysis of convergence of iterative methods and their acceleration. More recently, the first general circulation model was proposed over a Voronoi grid, and work continues in climate modeling utilizing Voronoi meshes and their duals [2]. In short, there exists a rich history of use of Voronoi (geodesic/hexagonal) grids in ocean and atmosphere modeling.

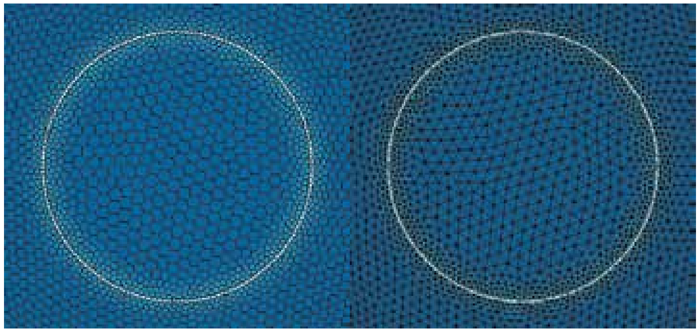

Fig. 1. A spherical centroidal Voronoi mesh and its Delaunay dual. The Voronoi cells straddle the interface while the triangle edges define it. Additionally, one can easily discern the increased density of cells around the interface.

Until recently, however, Voronoi-type unstructured grids came with certain limitations. One of these is global refinement in discrete steps, with sizes related to repeated dissection. Additionally, without both local and global refinement, there are issues such as interpolation error and artifacting, related to coupling multiple equations such as ocean and sea ice. To ameliorate the solution of the issues of coupling multiple equations over the sphere, we integrate a database of shoreline points into our grid generation process to demarcate various regions of the earth. Once we have sectioned off areas of the globe for different applications, we then add the ability to refine different sections based on arbitrary density proxies.

Methods exist that allow for fitting a triangulation to a boundary, but methods did not exist that allow for the use of multiple density proxies to be used to create unified, global grids suited to more than one application simultaneously. We utilize rich Voronoi grids that allow both global and local refinement in complementary ways. We create our grids using methods that allow for an arbitrary number of grid cells to be used, effecting global refinement to any degree desired, adding the ability to locally refine around shorelines, and to refine arbitrarily based on user input to better capture features of interest.

A spherical centroidal Voronoi tessellation (SCVT) is a special type of discretization of the sphere, defined by a set of points, generators, and a density function. These tessellations are generated by an iterative process. This iterative process allows us to tailor the grid by setting the initial conditions and the density function, leading to a very flexible system of mesh creation. Denoting a tessellation as Voronoi, along with its generators, implies that each region in the tessellation is composed of the subset of the area that is closer to the region's generator than any other. This Voronoi property provides for regions which are very regularly shaped. Adding centroidal as a qualifier to a Voronoi tessellation requires that each generator be the mass center of its region, and it is this property that requires iterative mesh creation. We use a custom density function to control the relative size of each region, as the density function is necessary in computing the centroids. Most importantly, a smoothly varying density function elicits a smooth change in region size that is, in general, a desirable property for nonuniform grids to have.

It is well known that the Delaunay triangulation and the Voronoi tessellation are planar duals of each other, thus the circumcenters of the triangulation are the vertices of the Voronoi regions and the mass centers of the Voronoi regions are the vertices of the triangulation. This

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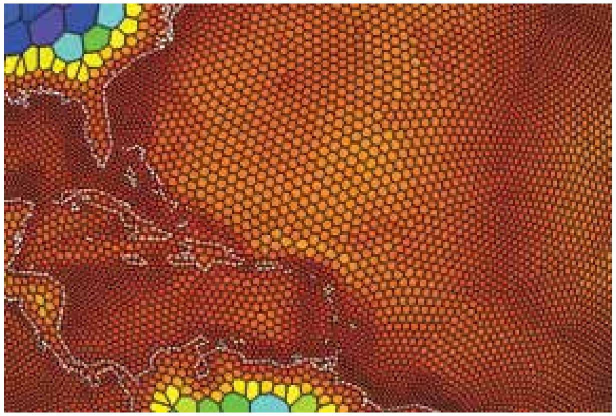

Fig. 2. A local view of a global SCVT, focused on the Florida coastline and the central Atlantic. The density is regulated by two disjoint proxies, bathymetry measurements in the ocean, and altimetry over the land. Note refinement along the shoreline interface.

means that, in our iterative methods, we can choose to compute either mesh and derive the dual directly from our iterative results. We use a Lloyd's iteration [3] to generate our SCVTs, described as: (1) start with a set of initial generators, (2) form a Voronoi diagram from the generators, (3) calculate the centroid of each Voronoi region using the density function, and (4) check for termination criteria, and, if not satisfied, return to step (2). In practice, we exploit the dual relationship between our meshes by forming a Delaunay triangulation and then producing the Voronoi connectivity from that by walking around the neighbors of each vertex in each triangle.

In order to create unified, unstructured grids that can service multiple equations at the same time, we must make use of the grid on both sides of the shoreline, and in order to size grid cells for multiple equations, we must be able to use an arbitrary number of density proxies. For example, we may wish to study the Gulf Stream, and need to refine the ocean grid in its area of influence around it, and simultaneously we may wish to refine for the Labrador Current. Another example might be to simultaneously refine a grid for an ocean model based on multiple criteria, such as sea surface temperature and potential vorticity. This is as simple as mapping the density proxy of interest, such as physical data, into the format used by our grid-generation software, and assigning it a bounding box to reside in. There are two main additions that we make to the iterative process that create our SCVTs: (1) a multiple proxy density function and (2) the concept of projecting points to the shoreline.

Shoreline (in general, boundary or interface) information is important when enabling multiple application codes to communicate over the interface of disjoint regions that abut. We use this accurate interface region to remove the hurdle of having separate grids whose cells adjacent to the shoreline do not match. When cells on the interface do not align, complicated procedures are required to communicate data from one application to the other, often increasing error or reducing convergence rates. We avoid these nonmatching interface region cells with our present technique. Additionally, we enforce a region of high density (i.e., small grid cells) in a band around shoreline interfaces. One could think of the band around the shoreline as being a safety net, for grid sizing, ensuring a safe maximum size around the shore, to provide for a smooth coupling of equations, and allowing the enforcement of grid size to safely relax inversely with the distance to the shore.

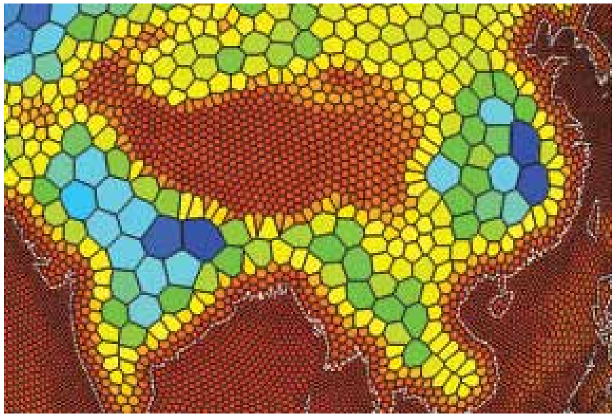

Fig. 3. A local view of a global SCVT, focused on the Himalayan mountain region. The density is regulated by two disjoint proxies, bathymetry measurements in the ocean and altimetry over the land. Note refinement along the shoreline interface.

In the balance, we increase the quality of multi-domain, multi-physics applications. We do this by enabling local refinement around interface regions that adjoin, and in application-specific regions of interest, while gaining arbitrary global refinement–all of these features are made feasible through Voronoi methods. The most practical examples are climate related and on the sphere, but all of our ideas translate readily to more general spaces and dimensions.

[1] Du, Q. et. al., *SIAM Rev* **41**(4), 637 (1999).

[2] Ringler, T.D. et. al., *Mon Weather Rev* **139**(11), 3348 (2011).

[3] Lloyd, S.P., *IEEE Trans Inform Theor* **28**(2), 129 (1982).